

# Superconductor/Insulator Transition in the Striped Phase

Yu. A. Dimashko and C. Morais Smith

*I Institut für Theoretische Physik, Universität Hamburg, D-20355 Hamburg, Germany*  
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We study the transversal dynamics of a charged stripe (quantum string) at  $T = 0$  and show that its kink excitations play the role of current carriers. If the hopping amplitude  $t$  is much smaller than the string tension  $J$ , the string is pinned and its ground state (GS) is insulating. At  $t \gg J$ , the string is depinned and the GS is a kink-condensate. By mapping the system onto a Josephson junction chain, we show that this state is superconducting. At  $(t/J)_c = 2/\pi^2$  the kink/antikink pairs decouple and a Kosterlitz-Thouless like insulator-superconductor transition occurs.

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The existence of a striped phase in doped 2D anti-ferromagnets (AF) has been recently a subject of intense experimental [1–3], numerical [4–6], and theoretical [7] investigations. Experimentally, elastic [1] and inelastic [2] neutron diffraction measurements in nickelates and cuprates have revealed the presence of charge and spin-order. Besides, muon spin resonance and nuclear quadrupole resonance results [3] have been also successfully interpreted within the picture of charged domain walls separating antiferromagnetic domains. Numerically, several studies reproduce this finding: in the framework of the one- or three-bands Hubbard model, or the simpler  $t - J$  model, exact diagonalization in small clusters [4], quantum Monte Carlo [5], and density matrix renormalization group [6] studies show the existence of the stripe configuration. Theoretical calculations based on the Hartree-Fock approximation [7] also point in the same direction.

The stripe configuration in doped AF can be regarded as an extended charge density wave (CDW). The conductivity from CDW was considered initially by Fröhlich [8], who showed the perfect conductivity of the collective (sliding) mode. The physical cause of this phenomenon is the blocking of the scattering processes by the Landau criterion [9]. Later, Lee, Rice, and Anderson (LRA) [10] analyzed the dynamics of the sliding mode in the presence of a lattice. They argued that pinning by the lattice and/or by impurities destroys the perfect conductivity. However, even pinned, the sliding mode gives rise to a large dielectric constant, as was experimentally observed in quasi 1D-compounds [11].

It is important to notice that LRA did not consider quantum fluctuations. This factor can compete with pinning even at  $T = 0$ . The question is, whether the quantum fluctuations are able to depin the stripe (or CDW), and whether this quantum depinning restores the perfect conductivity of the sliding mode.

In the present Letter, we investigate this question within a phenomenological model [12], which can be related to the  $t - J$  model [13]. We study the transversal dynamics of a single stripe, which is treated as a quan-

tum string. Then, by performing a canonical transformation in the quantum string Hamiltonian, we map the system onto a 1D array of Josephson junctions, which is known to exhibit an insulator/superconductor transition at  $(t/J)_c = 2/\pi^2$ . Further, we calculate the ground state (GS) of the quantum string in two limiting cases,  $t \ll J$  and  $t \gg J$ , and reveal the meaning of the transition in the “string” language. We argue that kink excitations of the string play the role of current carriers. At  $(t/J)_c$  the *insulating* pinned phase, corresponding to a GS with bound kink/anti-kink (K/AK) excitations turns into a *superconducting* depinned phase, where the K/AK pairs are decoupled and form a kink-condensate. In doing so, we have connected two important and different classes of problems, i.e., the transversal dynamics of stripes in doped AF and a system with well known superconducting properties. This suggests that phase coherence can be established within the stripes, providing us with an important clue for the understanding of the superconductivity mechanism in high- $T_c$  superconductors.

Let us consider a single vertical string on a  $N \times M$  square lattice (see Fig. 1a). The linear concentration of holes in the string is assumed to be one hole/site. The string is composed of  $N$  charged particles elastically interacting with the neighbour ones and constrained to move along  $N$  horizontal lines. The lattice constant is taken as the unit of length.

The classical state of the system is described by the  $N$ -dimensional vector  $\vec{x} = \{x_1, x_2, \dots, x_N\}$ . Here,  $x_n$  is the  $x$ -coordinate of the  $n$ -th particle,  $x_n = 1, 2, \dots, M$ . The corresponding quantum state  $|\vec{x}\rangle$  is defined as an eigenstate of all the coordinate operators  $\hat{x}_n$ ,  $n = 1, 2, \dots, N$ :  $\hat{x}_n |\vec{x}\rangle = x_n |\vec{x}\rangle$ . The phenomenological Hamiltonian describing this system is

$$\hat{H} = -t \sum_n (\hat{\tau}_n^+ + \hat{\tau}_n^-) + \frac{J}{2} \sum_n (\hat{x}_{n+1} - \hat{x}_n)^2. \quad (1)$$

The translation operators  $\hat{\tau}_n^\pm$  are defined by their action on the coordinate states,  $\hat{\tau}_n^\pm |\vec{x}\rangle = |\vec{x} \pm \vec{e}_n\rangle$ , where  $(\vec{e}_n)_m = \delta_{nm}$ . The coefficients  $t$  and  $J$  denote

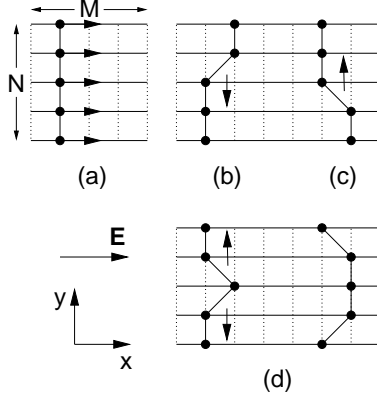


FIG. 1. Localized states of the quantum string and their translation modes induced by the electrical field  $\mathbf{E}$ : a) flat state; b) kink; c) antikink; d) pair kink/antikink.

the hopping amplitude and the string tension, respectively. The operators  $\hat{\tau}_n^\pm$  can be expressed through the momentum operators  $\hat{p}_n$ , which obey the canonical relation  $[\hat{x}_n, \hat{p}_m] = i\delta_{nm}$ . We then find  $\hat{\tau}_n^\pm = \exp(\pm i\hat{p}_n)$  and the Hamiltonian (1) becomes

$$\hat{H} = -2t \sum_n \cos \hat{p}_n + \frac{J}{2} \sum_n (\hat{x}_{n+1} - \hat{x}_n)^2. \quad (2)$$

Hereafter, we classify the state of the quantum string by the value of the topological charge  $\hat{Q} = \sum_n (\hat{x}_{n+1} - \hat{x}_n)$ . In the case of open boundary conditions (BC), the topological charge is an arbitrary integer,  $Q = 0, \pm 1, \pm 2, \dots$ . The states with positive and negative charges are called kinks (K) and antikinks (AK), respectively (see Figs. 1b and 1c). Here, we consider periodic BC,  $\hat{x}_{N+1} = \hat{x}_1$ . Hence, the total topological charge of the string is zero (Figs. 1a and 1d).

Since we are interested in the conducting properties of the system, we have to determine the current operator  $\hat{j}_n = e\dot{\hat{x}}_n$ , where  $e$  is the charge of the particle and the dot denotes the time derivative. Using the equation of motion  $\dot{\hat{x}}_n = i[\hat{H}, \hat{x}_n]$ , we obtain  $\hat{j}_n = 2et \sin \hat{p}_n$ .

At this point, it is convenient to perform a dual transformation to new variables referring to the segments of the string, i.e., to a pair of neighbour holes,

$$\hat{x}_n - \hat{x}_{n-1} = \hat{\pi}_n, \quad \hat{p}_n = \hat{\varphi}_{n+1} - \hat{\varphi}_n. \quad (3)$$

The new local variables also obey the canonical relation  $[\hat{\varphi}_n, \hat{\pi}_m] = i\delta_{nm}$ . Furthermore, we take the limit  $M \rightarrow \infty$  in order to deal with all operators in the  $\varphi$ -representation,  $\hat{\varphi}_n \Rightarrow \varphi_n$ ,  $\hat{\pi}_n \Rightarrow -i\partial/\partial\varphi_n$ . The continuous variable  $\varphi_n$  is restricted to the interval  $0 \leq \varphi_n < 2\pi$ . Finally, the Hamiltonian and the current operator acquire the form

$$\begin{aligned} \hat{H} &= -2t \sum_n \cos(\varphi_{n+1} - \varphi_n) - \frac{J}{2} \sum_n (\partial/\partial\varphi_n)^2, \\ \hat{j}_n &= 2et \sin(\varphi_{n+1} - \varphi_n), \end{aligned} \quad (4)$$

which is known from the theory of superconducting chains. Eqs. (4) describe a Josephson junction chain, with the Coulomb interaction taken into account. The solution of this problem at  $T = 0$  has been found by Bradley and Doniach [14]. Depending on the ratio  $t/J$ , the chain is either insulating (small  $t/J$ ) or superconducting (large  $t/J$ ). The results arise from the standard mapping of the 1D quantum problem onto the 2D classical one. In this way, one obtains the XY model with Euclidean action

$$\mathcal{S}_E = \sqrt{\frac{2t}{J}} \sum_{\langle \vec{r}, \vec{r}' \rangle} \cos(\varphi_{\vec{r}} - \varphi_{\vec{r}'}), \quad (5)$$

where the vectors  $\vec{r} = (n, \tau)$  form a rectangular lattice in space and imaginary time.

At  $t/J = 2/\pi^2$  the Josephson chain undergoes a Kosterlitz-Thouless (KT) transition. For small  $t/J$  values, the two-points correlator  $\langle \exp i(\varphi_{\vec{r}} - \varphi_{\vec{r}'} ) \rangle$  decays exponentially. Then, the frequency dependent conductivity exhibits a resonance,  $\text{Re } \sigma(\omega) \propto \delta(\omega - J)$ . Since there is no conductivity at  $\omega = 0$ , this is an insulating state with a gap  $\Delta = J$ . In the opposite case, when  $t/J$  is large, the same correlator decays algebraically. Then, the conductivity is singular at  $\omega = 0$ ,  $\text{Re } \sigma(\omega) = 2\pi e^2 t \delta(\omega)$ , and the system is superconducting.

These results are also valid for the quantum string on the lattice, since both systems are described by exactly the same Hamiltonian and current operator, see Eqs. (4). Now, it remains to reveal the physical significance of these results for the striped phase. Mainly, it is important to understand the origin of the current and the meaning of the KT transition in the “string language”. In order to achieve this aim, it is instructive to analyse the problem in two limiting cases, which allow for an explicit solution:  $t \ll J$  and  $t \gg J$ .

Let us start considering the limit of weak fluctuations, with  $t \ll J$ . In the absence of hopping,  $t = 0$ , the excitation spectrum of the string is discrete,  $E_\nu = \nu J$ , with  $\nu = 0, 1, 2, \dots$ . The ground level  $E_0 = 0$  corresponds to a flat state of the string, i.e., to the kink-vacuum  $|0\rangle$  (see Fig. 1a). The first elementary excitation corresponds to the creation of a pair K/AK (see Fig. 1d), i.e., to the state  $\hat{A}_n^+ \hat{A}_m^- |0\rangle$ . Here,  $\hat{A}_n^\pm = \exp(\pm i\hat{\varphi}_n)$  are local creation operators of K(+) and AK(-). This state has energy  $E_1 = J$ . All the levels are degenerated, and therefore, accounting for small but nonzero hopping  $t$  can split them.

The ground level  $E_0$  splits only in the  $N$ -th order of the perturbation theory. Indeed, any flat state of the string can be transformed into another flat one only after  $N$  elementary translations (see Fig. 1a). Hence, the ground level  $E_0$  acquires a width  $\propto J(t/J)^N$ , which tends to zero in the thermodynamical limit  $N \rightarrow \infty$ . In this limit the ground level is *not* split.

The first excited level  $E_1$  splits already in the first

order of the perturbation theory. Resolving the secular equation, one finds the energy  $E_1(k, q) = J - 2t(\cos k + \cos q)$  and the corresponding states

$$|k, q\rangle = \frac{1}{\sqrt{2}}(\hat{A}_k^+ \hat{A}_q^- - \hat{A}_q^+ \hat{A}_k^-) |0\rangle. \quad (6)$$

Here,  $A_k^\pm = (1/\sqrt{N}) \sum_n \exp(\pm i\varphi_n + ikn)$  are creation operators of K/AK with fixed momentum  $k$ .

Thus, the level  $E_1$  acquires finite band width  $\simeq 8t$ . Actually, one can show that each  $\nu$ -th level splits into a band of width  $\simeq 8\nu t$ . The energy band structure of the quantum string is shown in Fig. 2. The excitation spectrum has a gap  $\Delta \simeq J - 4t$ , which is nothing but the minimal energy required to create a K/AK pair. The existence of the gap at  $t \ll J$  suggests that the ground state (GS) of the string in this limit is insulating.

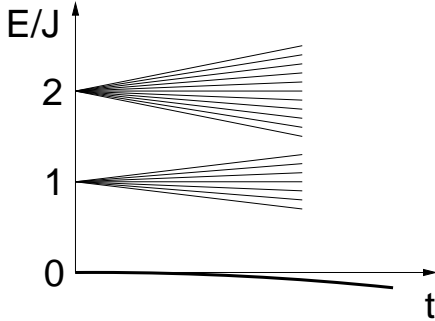


FIG. 2. Energy spectrum of the quantum string at small nonzero hopping,  $t \neq 0$  and  $t/J \ll 1$ . The ground level is not split, but it decreases as a quadratic power-law.

Nevertheless, it is interesting to consider the conductivity of the *excited* states caused by the presence of the kinks. In the excited state (6) we have one unbound pair K/AK. They can move in opposite  $y$ -directions. This leads to an effective one-step motion of the string in the  $x$ -direction, with the consequent appearance of current (see Fig. 1d). Thereby, the kinks play the role of current carriers. By averaging the current operator  $\hat{I} = \sum_n \hat{j}_n$  over the K- and AK-states  $\hat{A}_k^\pm |0\rangle$ , we find  $\langle \hat{I} \rangle = \mp 2et \sin k$ , which is similar to the current  $I = 2et \sin k$  for one free hole. However, in contrast to the hole, the effective electrical charge of the kink is determined by its topological charge. This fact can also be seen from a different analysis. Let us apply an electrical field  $E$  along the  $x$  axis (see Fig. 1). Then, a one-step shift of the K (AK) in the  $y$ -direction,  $y \rightarrow y + 1$ , results in a energy change of  $eE$  ( $-eE$ ). Hence, one can ascribe an effective electrical charge  $-e$  ( $+e$ ) to the K (AK).

It is worth to note that in the  $t \ll J$  limit the GS has only local K/AK pairs. A perturbative calculation of the K/AK pair correlation function reveals only short-range correlations  $\langle \hat{A}_n^+ \hat{A}_m^- \rangle = \langle \exp i(\varphi_n - \varphi_m) \rangle \propto \exp(-|n-m|/R)$ . The correlation length  $R = 2/\ln(J/2t)$

can be treated as an average dimension of the virtual K/AK pairs. The local K/AK pair cannot carry current because it forms a bound state with zero topological charge. Thereby, the locality of the K/AK pairs is responsible for the insulating character of the GS at small  $t$ .

Now, we concentrate on the opposite limit, when the fluctuations are strong and  $t \gg J$ . In this case, we can expand the  $t$ -term in the Hamiltonian (4) up to second order,  $\cos(\varphi_{n+1} - \varphi_n) \approx 1 - (\varphi_{n+1} - \varphi_n)^2/2$ , and diagonalize the quadratic Hamiltonian. Then, we obtain the phonon-like spectrum  $E_k = -2tN + \omega_k$ ,

$$\omega_k = \sqrt{8tJ} |\sin(k/2)|, \quad (7)$$

with a finite band width  $\sqrt{8tJ}$  and no gap. Therefore, the GS is conducting. Calculations of the conductivity are straightforward, since in this case, the time dependence of the current  $\hat{j}_n \approx 2et(\hat{\varphi}_{n+1} - \hat{\varphi}_n)$  follows from the standard relation,

$$\hat{\varphi}_n(\tau) = \sum_k \sqrt{\frac{J}{2N\omega_k}} [e^{i(kn - \omega_k\tau)} \hat{a}_k + e^{i(\omega_k\tau - kn)} \hat{a}_k^\dagger]. \quad (8)$$

Here,  $\hat{a}_k$  and  $\hat{a}_k^\dagger$  are Bose operators. Using these expressions, we calculate the current-current correlator

$$\Pi(k, \omega) = -i \int_0^\infty d\tau e^{i\omega\tau} \langle [\hat{j}_k^\dagger(\tau), \hat{j}_k(0)] \rangle \quad (9)$$

and the uniform conductivity

$$\sigma(\omega) = -\frac{1}{\omega} \lim_{k \rightarrow 0} \text{Im} \Pi(k, \omega) = 2\pi e^2 t \delta(\omega) \quad (10)$$

Furthermore, the K/AK pair correlator exhibits quasi-long range order,  $\langle \hat{A}_n^+ \hat{A}_m^- \rangle = \langle \exp i(\varphi_n - \varphi_m) \rangle \propto |n-m|^{-\gamma}$ , with  $\gamma = \sqrt{J/8\pi^2 t}$ . Hence, in the limit  $t \gg J$  the average dimension of the K/AK pairs diverges (the pairs decouple), providing the conducting GS. However, this does not mean the absence of the correlations. Controversally, the GS exhibits quasi-long range correlations of the phase.

The most important property of the GS in this limit is the presence of decoupled kinks and antikinks, which form a new kind of correlated state: a *kink-condensate*. Another property of the kink-condensate is its gauge invariancy. Hamiltonian (4) possesses the gauge symmetry ( $\varphi_n \rightarrow \varphi_n + a$ ), which is generated by the topological charge operator  $\hat{Q} = -i \sum_n \partial/\partial \varphi_n$ . Since the kink-condensate is the single GS, it is gauge-invariant. This can be also seen from the fact that the total topological charge is  $Q = 0$ . The phonon-like excitations (7) break the gauge symmetry and do not break the translational one ( $x_n \rightarrow x_n + a$ ). They obey the Landau criterion [9], providing superflow and supercurrent for sufficiently small velocities of the condensate  $v < v_0 = (2/\pi)\sqrt{2tJ}$ .

In this way, the superconductivity of the quantum string is related to the existence of the kink-condensate.

It is important to notice that in the case of the Josephson chain, the variable  $\varphi_n$  is a single-valued function of  $n$ . In order to keep it single-valued after the dual transformation (3), one has to require the total momentum of the string to be zero,  $P = \sum_n p_n = 0$ . Otherwise, the periodic condition  $\varphi_{N+1} = \varphi_1$  cannot be satisfied and the phase becomes multivalued. Formally, the restriction  $P = 0$  excludes uniform current states of the string. However, the thermodynamical limit  $N \rightarrow \infty$  allows to treat the uniform conductivity as a long-wave limit  $k \rightarrow 0$ , see Eq.(10). On the other hand, the uniform current state describing motion of the condensate with velocity  $v$  can be generated by applying the operator  $\exp[i(v/2t) \sum_n \hat{x}_n]$  on the GS of the condensate [8].

Finally, we can summarize our results: at  $t = 0$ , the GS of the string is the kink-vacuum. At  $0 < t/J \ll 1$ , it has only bound K/AK pairs, exhibits no long-range phase order, and the energy spectrum is gapped. Then the system is insulating. At  $t/J \gg 1$ , the K/AK pairs are already decoupled and there is no gap anymore. Then the phase is quasi long-range ordered, the GS is the kink-condensate and the system is superconducting. Thus, our calculations in both limiting cases are in agreement with the results obtained from the mapping onto the Josephson chain, with the advantage that they clarify the physical meaning of the insulating and superconducting states for the quantum string.

Based on the Josephson chain results, it follows that at  $(t/J)_c = 2/\pi^2$  the quantum string undergoes a KT-transition. This transition has been qualitatively predicted by Eskes *et al.* [12], and treated as roughenning of the string. Besides, Viertiö and Rice [15] have calculated the energy for creating a pair K/AK and have shown that for large  $t/J$  values this energy becomes negative, leading to a proliferation of K/AK pairs. Here, we have shown that at the transition point the gap  $\Delta$  vanishes and the K/AK pairs decouple. However, this decoupling means not only roughenning of the string, but also *quantum depinning* from the lattice and *superconductivity* of the rough depinned phase. The quantum fluctuations turn out to be able to depin the string from the lattice and to restore the Fröhlich's perfect conductivity. Although our conclusions are based on the single stripe picture, we expect the results to remain valid at higher (but not too high) doping concentrations, as well as in the presence of impurity pinning. Indeed, as shown in Ref. [16], by accounting for both factors, a "free phase" arises within a certain parameter range. Actually, we show that this "free phase" is superconducting.

We want to emphasize that at finite temperatures ( $T \neq 0$ ), thermal fluctuations will "spoil" the superconductivity for both, the Josephson chain and the quantum string. In this case, the Euclidean action (5) describes a XY model on a 2D lattice, which is finite in

the  $\tau$ -direction, with length  $L = 2\pi/T$ . Then, the KT-transition disappears and the long-range phase correlations, as well as superconductivity, are suppressed. This problem is rooted in the 1D treatment of the dynamics. Since we are considering a filled stripe and accounting only for transversal fluctuations, superconductivity cannot take place, except at  $T = 0$ . This could explain why the nickelates never become superconducting. At higher dimensions, thermal fluctuations do not play such a destructive role anymore. Another problem is the Meissner effect and flux quantization. They cannot be verified in the 1D model, due to the absence of non-zero volume and of small contours, respectively. In 1D there is no difference between superconductivity and perfect conductivity. Therefore, a 2D theory, coupling the longitudinal and transversal dynamics in a half filled stripe seems to be the next step needed for gaining some insight about superconductivity in the cuprates.

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